

# THREE DIMENSIONAL MODELING OF THE LEFT VENTRICLE OF THE HEART USING SPHERICAL HARMONIC ANALYSIS

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## ABSTRACT

Myocardial scintigraphy SPECT (Single Photon Emission Computed Tomography) is a functional imaging modality which is performed at stress and rest. The diagnosis is obtained by comparing myocardium blood flow at these two different patient states. We propose to add at this technique completely non invasive anatomical data to avoid the use of invasive modalities like coronarography for example. For this purpose, we intend to extend the powerful technique of 2D Fourier descriptor to 3D objects by modeling the left ventricle at stress and at rest using the spherical harmonic descriptors so as to provide quantitative information to the physician to evaluate the extent of an eventual ischemia.

**Index Terms**— Myocardial scintigraphy, Left ventricle, Spherical parametrization, Invariant descriptors, Spherical harmonics

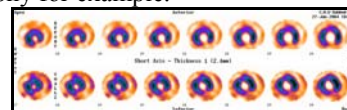
## 1. INTRODUCTION

Advanced techniques of cardiac imagery provide today better assessment of the left ventricle (LV) shape and function and allow visualization of its outer and inner walls (epicardium and endocardium) with increasing resolution in space and time. Consequently, data available is large and unstructured. That is why it is important to find a pertinent model to help the physician to establish a reliable diagnosis. Extensive researches have been conducted in order to reconstruct and model the LV geometry. A global representation for example was used to describe the surface as a whole. It reduced a shape as a set of parameters where each one affects the entire shape. Staib and Duncan [7] used a torus topology. They applied this approach on many shapes found in medical imagery. However it illustrates some difficulties especially in representing shapes with spherical topology. Algebraic surfaces are also used in modelling LV shape. Cauvin and al. [3] introduced a half ellipsoid in apical region with a cylinder in the basal one. Superquadrics are a rich set of surfaces that can represent a large number of real and medical objects. Bardinet and al. [1] present an approach based on these surfaces to analyse the shape and the deformation of the LV of the heart.

Spherical harmonic analysis is very much analogous to Fourier shape approach. The real difficulty comes from surface parametrization. Brechbuhler and al. [2] present a method of parametrization of closed surfaces for 3D shape description. The parametrization is formulated as a constrained optimization problem. The convergence of this program becomes unstable for object meshes consisting of several thousand vertices. Authors in [5] apply a new optimization process which aims to minimize distortions that can appear after mesh projection to the parameter space. In this paper, we describe a one to one mapping to the sphere and a uniform parametrization that aims to optimize the model reconstruction and to overcome limitations caused by modelling non star-shaped objects. Myocardial scintigraphy is used to evaluate this approach so as to give anatomical information to these functional and non invasive cardiac images. Reconstructed models of the LV at stress and at rest are compared to quantify deformation between these two particular sequences.

## 2. MYOCARDIAL SCINTIGRAPHY

Myocardial scintigraphy with thallium 201 (*Figure 1*) is a simple, safe, and valuable non invasive functional technique in evaluating the condition of patients with cardiac disorders. Images are obtained at rest and stress. The diagnosis of an ischemic heart disease is obtained by comparing the topology of myocardium blood flow at these two different instants. In this work, we aim to add to these functional information anatomical data which can help the physician to evaluate numerically the extent of an ischemia. This can avoid the use of invasive modalities like coronarography for example.



*Fig 1:* Myocardial scintigraphic data - the first line: sequence at stress, second line: sequence at rest.

For this purpose, segmentation of scintigraphic 2D layers must be performed to extract epicardium data. A segmentation based on the histogram of the intensities is used to isolate grossly the LV cavity, and therefore, with the

help of mathematical morphology operators [6] we automatically smooth and isolate both surfaces. Reconstruction of both endocardium and the epicardium walls (Figure 2) provides information about the thickness of the myocardium muscle by making longitudinal cuts in the rendered surface.

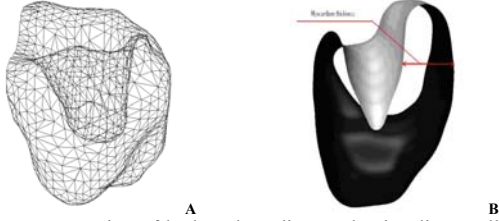


Fig 2: 3D representation of both endocardium and epicardium walls – A: Triangular wireframe, B: rendered surface of a longitudinal cut of A (Visualization of the myocardium thickness)

Triangular meshes obtained after reconstruction are unstructured and consisting of several thousand vertices. So it is necessary to find a pertinent and compact model that we can manipulate easily. One of the principle challenges faced in the area of shape modeling is that a model and its image under a transformation are considered to be the same. The challenge in comparing two shapes is to find a metric between these shapes. In this paper, we use the spherical harmonic analysis to model the epicardium wall because it is a general tool that can be applied to many kinds of shapes especially genus zero surfaces and it provides a reduction in both storage space and time of comparison.

### 3. SPHERICAL HARMONIC ANALYSIS

LV surface obtained after triangulation is a genus zero object which could be represented by a spherical bidimensional parametrization. In this case, spherical harmonic analysis is applied to derive a complete and invariant shape representation. By completeness, representation contains sufficient information for reconstructing the original object. Invariance is a property of geometric configurations which remained unchanged under an appropriate class of transformations.

#### 3.1. Fourier transform on a group

A representation of a given group  $G$  is a continuous operator  $T$  on this group, taking values in the group of non singular continuous linear transformations of the linear space  $V$  and satisfying the functional equation :

$$T(g_1 g_2) = T(g_1) T(g_2) \quad (1)$$

Let  $f$  be in  $L^1(G, d\mu)$ , where  $G$  is assumed to be abelian and  $\mu$  is the invariant measure of  $G$ . The Fourier transform on  $G$  is defined by :

$$TF(f) = \int_G f(x) [T_\lambda(x)]^{-1} d\mu(x) \quad (2)$$

Where  $T$  represents all irreducible and unitary representation of  $G$ .

In our case, LV surfaces are considered to be genus zero. Thus,  $f$  is defined on  $S^2$  which is compact. The group of 3D Euclidean motion  $M(3)$  is the cross product of translations isomorphic to  $IR^3$  and the group of the 3D rotation  $SO(3)$  :  $M(3) = IR^3 \times SO(3)$  [4]. Consequently, after the spherical parametrization, the invariance formulation can be schematized as follow:

$$(R^3 \times SO(3) \times S^2) \times L_{R^3}^2(S^2) \rightarrow L_{R^3}^2(S^2) \quad (3)$$

$$(B, A, (\varphi_0, \theta_0)), f \rightarrow Af(\varphi + \varphi_0, \theta + \theta_0) + B$$

Where  $L_{R^3}^2(S^2)$  denotes the space of functions  $f$  on  $S^2$  verifying :

$$\|f\|^2 = \int_{S^2} |f(\xi)|^2 d\xi < \infty \quad (4)$$

and  $d\xi$  is the normalized invariant measure on  $S^2$  given by

$$d\xi = \frac{1}{4\pi} \sin \theta d\theta d\varphi$$

The compactness property of  $S^2$  implies that the Fourier transform exists and is discrete. It corresponds on the Fourier coefficients calculated on the well known basis of  $L_{R^3}^2(S^2)$  noted by  $e_l^m$  which is expressed according to Legendre associated functions. By applying Fourier coefficients on  $S^2$ , the shift theorem transforms (3) on:

$$(R^3 \times SO(3) \times S^2) \times L_{C^3}^2(Z \times Z) \rightarrow L_{C^3}^2(Z \times Z) \quad (5)$$

$$(B, A, (\varphi_0, \theta_0)), \{a_l^m(f)\} \rightarrow e_l^m(\varphi_0, \theta_0) A a_l^m(f) + B$$

#### 3.2. Spherical parametrization

To apply spherical harmonic analysis, it is important to find a spherical parametrization that allows the mapping onto the  $S^2$ . For 3D objects, parametrization is far less obvious because surfaces cannot be traced in an equally simple manner as can be done for the contour of 2D region.

We present a uniform one to one mapping onto the parameter space  $S^2$  that can be used even for non star shaped objects. By uniformity, we intend to obtain an equally distributed points on the sphere which can approach the mathematical formulation given by (3).

##### 3.2.1. Mapping onto the sphere

The method we use to map the vertices from the original object to the sphere surface is an iterative blow-up algorithm in which the inflation of a balloon is simulated. The centre position of the target sphere is the same that the object one. The sphere radius  $R_s$  is fixed at the beginning of the mapping process which converges when all vertices of the initial mesh are projected onto the sphere.

$$R_s = 2 \max_i |v_i^p| \quad (6)$$

Where  $|\vec{v}_i|$  is the vertex radius of the  $i^{th}$  vertex.

During each iteration, vertices move toward the parameter space. This migration depends on vertices position. The further are the vertices are far from the sphere centre, the faster the projection is. This process is formulated by:

$$R_i^{new} = R_i^{old} + a \frac{|R_s - R_i^{old}|}{R_s} * (R_s - R_i^{old}) \quad (7)$$

Where  $R_i^{new}$  is the new radius calculated in the  $i^{th}$  iteration,  $R_i^{old}$  is the old radius of the  $(i-1)^{th}$  iteration and  $a$  is a constant.

### 3.2.2. Uniformization

Uniformization is added for minimizing errors and distortions due to parameterization. In fact, when points of the parametrized mesh are equally distributed on the surface, distortions and errors are reduced considerably. Experimental demonstration of the importance of the uniform parametrization is given in [2].

The uniform mesh method is based on vertices area equalization. The objective of this approach is to obtain, through an iterative process, a triangular mesh composed of equal area faces. At each iteration of the mapping process, the Cartesian coordinates of each vertex are altered in order to equalize the adjacent faces. This procedure is formulated by:

$$\vec{x} = \frac{\sum_{k=1}^K A_k \vec{\mathcal{E}}_k}{\sum_{k=1}^K A_k} \quad (8)$$

With  $\vec{x}$  is the spatial coordinate vector of a vertex,  $K$  is the number of the surrounding faces,  $A_k$  is the area of the  $k^{th}$  triangle and  $\vec{\mathcal{E}}_k$  is the spatial coordinate vector of its centre.

### 3.3. Rotation invariant descriptors

Using the formulation presented at (5) we can extract pertinent invariant descriptors under  $SO(3)$ . We propose the following set (\* : transpose and conjugate operator):

$$I_l^m(f) = \sum \sum a_l^m(f) a_l^m(f)^* \quad (9)$$

Consequently, comparison between spherical harmonic models can be performed by calculating the  $L^2$  difference between the invariant descriptors of  $f$  and  $f'$  formulated by :

$$D^2 = \sum_{l=0}^{+\infty} \sum_{m=-l}^l (I_l^m(f) - I_l^m(f'))^2 \quad (10)$$

## 4. EXPERIMENTAL RESULTS

In this section, we illustrate and discuss the main results obtained on synthetic and medical images.

### 4.1. Synthetic images

In order to validate the parametrization method, we start by apply the approach presented above on a synthetic meshes.

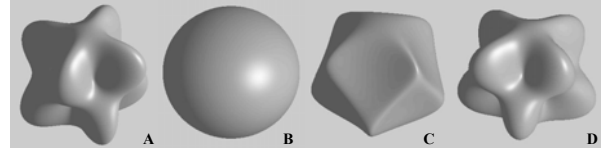


Fig 3: Reconstruction of spherical model - A: initial synthetic object, B: degree 1, C: degree 5, D: degree 6

### 4.2. Medical images

We illustrate, in this section, projection steps from the initial object (epicardium surface) to the parameter space (the unit sphere (Figure 4)). At each iteration, we try to uniform the projected mesh by adjusting triangular areas.

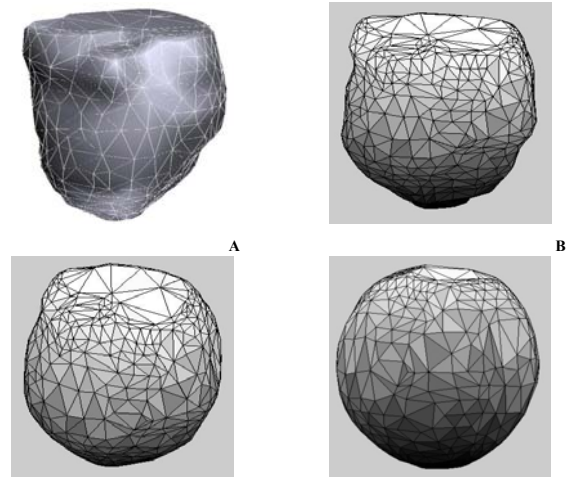
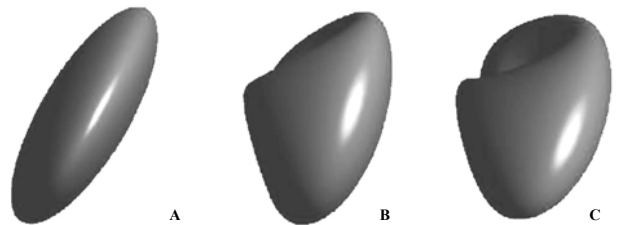


Fig 4: Spherical parametrization - A: Epicardium wall of a patient at rest, B: 3<sup>rd</sup> iteration, C: 30<sup>th</sup> iteration, D: 100 iteration

#### 4.2.1. Spherical model reconstruction

After extraction of spherical harmonic coefficients, the reconstruction of the LV model can be performed (eq. 2). It is important to add that these coefficients are global and pertinent. That is meaning that with a small number of spherical harmonics (low frequencies) we can represent the global form of the LV of the heart. In Figure 5, we illustrate reconstructions steps up to the 7<sup>th</sup> degree of the epicardium wall shown in Figure 4-A.



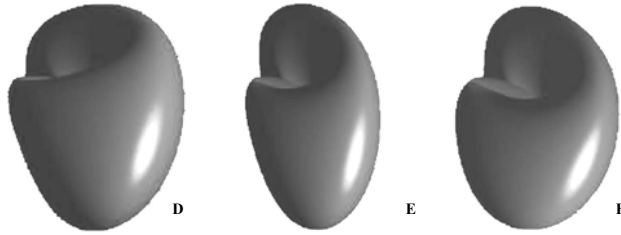


Fig 5: Reconstructed models using spherical harmonic series truncated at different degrees - A: degree 1, B: degree 2, C: degree 3, D: degree 4, E: degree 5, F: degree 7

### 5.2.2. Quantitative analysis of the LV between stress and rest

Modelling the LV surface at stress and rest by two harmonic surfaces allows a numerical estimation of the deformation through these two specific states (Eq. 6). This evaluation can help the physician to make a reliable diagnosis. For the two following figures (Figure 6 and 7), we have reconstructed spherical harmonic models from scintigraphic data of one healthy voluntary and a pathologic case respectively. Figure 6 shows that the distance between the two reconstructed models is valued as 0.26. Figure 7 shows that the distance value calculated is 1.05.

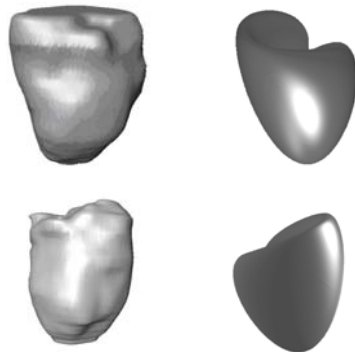


Fig 6: Modelling of the epicardium wall of a voluntary – Left: rendered surfaces respectively at stress and rest, right: reconstructed models using spherical harmonics respectively at stress and at rest

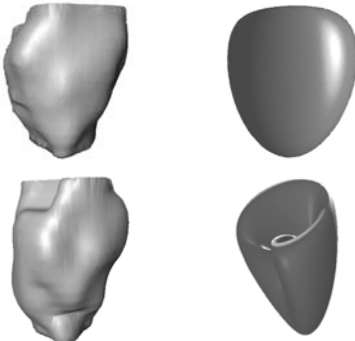


Fig 7: Modelling of the epicardium wall of a patient – Left: rendered surfaces respectively at stress and rest, right: reconstructed models using spherical harmonics respectively at stress and at rest

These numeric values are additional information obtained from scintigraphic data and tell the patrician about the extension of the disease. In fact, distances can be ordered and grouped by intervals. Each one represents a specific degree of the cardiac pathology.

## 6. CONCLUSION

We have proposed in this paper an approach to model anatomical structures particularly the LV of the heart using spherical harmonic descriptors. This approach is based on a mapping toward the sphere and a uniform parametrization. The obtained model is compact and pertinent and allows us to evaluate deformation between two specific states, stress and rest, by calculating distance separating the two respective models. This comparison is one proposed technique to extract quantitative parameters and to evaluate the extension of cardiac pathology. To validate our approach, we applied first the three dimensional modelling process on synthetic data and then we used scintigraphic images which is an imaging modality providing functional information in order to give supplement anatomical aspect and to help patricians to make reliable diagnosis. Future directions will include a mathematical method to avoid the appearance of distortions in high frequencies during the model reconstruction step. A more efficient algorithm will be proposed to obtain a better uniform parametrization.

## 7. REFERENCES

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